6. K. Stewartson, "Falkner–Skan equation for wakes," AIAA J., 2, No. 7 (1964).
7. M. Abramovits and I. Stegun (eds.), Handbook of Special Functions [Russian translation], Nauka, Moscow (1979).
8. A. Acrivos, D. D. Snowden, et al., "The steady separated flow past a circular cylinder at large Reynolds numbers," J. Fluid Mech., 21, Pt. 4 (1965).
9. A. Acrivos, L. G. Leal, et al., "Further experiments on steady separated flows past bluff objects," J. Fluid Mech., 34, Pt. 1 (1968).

HEAT TRANSFER IN A FILM FLOWING OVER THE SURFACE OF A CONVERGENT DUCT

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Efficient organization of the process of thermal softening of highly-mineralized natural liquids such as sea water requires their heating to temperatures above 200°C with a nonboiling regime of operation of the heat-transfer unit. One method of realizing such heating is the use of film-type units, in which heat is supplied to a laminar film of liquid from the free phase boundary [1]. In contrast to recuperative heat exchange, in this case the minearalized liquid can be heated to a high temperature while the temperature of the boundary layer of the film is relatively low. It is this circumstance that permits non-boiling operation of the water heater.

It has been established experimentally [2] that the flow of fluid in a convergent duct with a total convergence angle of more than 90° (in contrast to flow over a vertical surface) results in a two-dimensional laminar nonwavy regime of film flow with a broad range of flow rates. This hydrodynamic feature makes it possible to more fully utilize the advantages of the given method of heating and accounts for the preference of using convergent-duct-film units [3, 4] to heat scale-forming solutions.

Here we study the process of contact heat exchange in the condensation of pure vapor on a film of liquid flowing over the surface of a convergent duct.

Formulation of the Problem. Assuming the problem to be steady and axisymmetric, we write the following equations of conservation of momentum, continuity, and energy in a boundary-layer approximation for a thin liquid film:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)+g\sin\alpha; \tag{1}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y}+g\cos\alpha=0; \tag{2}$$

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0; \tag{3}$$

$$\rho c\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right). \tag{4}$$

Here, x is the longitudinal coordinate, directed downflow along the generatrix of the convergent duct; y is the transverse coordinate, directed perpendicular to the generatrix of the duct; the origin of the coordinates is on the inlet edge of the duct; u and v are respectively the x and y components of velocity; g is acceleration due to gravity; p is pressure; α is the angle of inclination of the duct generatrix to the horizontal; r(x) is the running radius of the duct; ν, ρ, c, λ, and T are the kinematic viscosity, density, specific heat, thermal conductivity, and temperature of the liquid.

The problem is solved with the following assumptions: the subjacent surface of the duct is thermally insulated; there is no shear stress on the liquid-vapor boundary; the

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condensate of the vapor has no effect on flow or heat transfer. In this case, the boundary conditions for (1)-(4) are represented in the form

$$\partial T/\partial y = 0, \; u = v = 0 \quad \text{at} \quad y = 0; \tag{5}$$

$$T = T_h, \; \partial u/\partial y = 0, \; u\partial h/\partial x = v, \; p = p_h \quad \text{at} \quad y = h(x); \tag{6}$$

$$T = T_0, \; h = h_0 \quad \text{at} \quad x = 0, \tag{7}$$

where h_0 is the thickness of the film at the inlet of the duct; T_0 and T_h are the temperature of the film at the inlet and the saturation temperature at the phase boundary; p_h is the vapor pressure above the film surface.

Assuming that the velocity profile in the film is self-similar

$$u(x, y) = \langle u\rangle(x) f(z), \quad z = y/h(x), \quad \int_0^1 f(z)\,dz = 1 \tag{8}$$

and integrating (1) and (3) through the film thickness with the use of (2) and (6), we obtain the dimensionless equations [5]

$$\left[1 - (1 - X)^2 H^3 \frac{\cos\alpha}{\gamma\,\mathrm{Fr}}\right]\frac{dH}{dX} = \frac{H}{1 - X} + \frac{\beta(1 - X)}{\varepsilon\gamma\,\mathrm{Re}} - \frac{\sin\alpha}{\varepsilon\gamma\,\mathrm{Fr}}(1 - X)^2 H^3; \tag{9}$$

$$VH(1 - X) = 1. \tag{10}$$

Here $X = x/l_0$; $H = h/h_0$; $\mathrm{Fr} = \langle u_0\rangle^2/(gh_0)$; $\mathrm{Re} = \langle u_0\rangle h_0/\nu$ (ν is taken at the temperature of the wall); $V = \langle u\rangle/\langle u_0\rangle$; $\varepsilon = h_0/l_0$ (Eq. (9) is valid at $\varepsilon \ll 1$); $\beta = df/dz|_{z=0}$; $\gamma = \int_0^1 f^2(z)dz$; l_0 is the length of the generatrix of the duct; $\langle u_0\rangle$ is the mean velocity at the inlet.

We express Eqs. (3) and (4) in the variables $x \sim z$:

$$\frac{\partial(rv)}{\partial z} - \frac{\partial(ruz)}{\partial z}\frac{dh}{dx} + \frac{\partial(rhu)}{\partial x} = 0; \tag{11}$$

$$h^2\rho c\left[u\frac{\partial T}{\partial x} + \left(\frac{v}{h} - u\frac{z}{h}\frac{dh}{dx}\right)\frac{\partial T}{\partial z}\right] = \frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right). \tag{12}$$

When condition (10) is satisfied, the term in parentheses in the left side of Eq. (12) is identically equal to zero. In fact

$$\frac{\partial(rh\langle u\rangle)}{\partial x} = l_0 h_0\langle u_0\rangle\cos\alpha\frac{\partial}{\partial x}[HV(1 - X)] \equiv 0.$$

As a result, by using (11) we obtain

$$\frac{v}{h} - u\frac{z}{h}\frac{dh}{dx} = -\frac{1}{rh}\left[-(rv) + ruz\frac{dh}{dx}\right] = -\frac{1}{rh}\left(\int_0^z f(z)\frac{\partial(rh\langle u\rangle)}{\partial x}dz - ruz\frac{dh}{dx} + ruz\frac{dh}{dx}\right) \equiv 0.$$

With allowance for (8) and (10), we represent (12) in dimensionless form

$$C_V\,\mathrm{Pe}\,\varepsilon\frac{Hf(z)}{1 - X}\frac{\partial\Theta}{\partial X} = \frac{\partial}{\partial z}\left(K\frac{\partial\Theta}{\partial z}\right). \tag{13}$$

Here, $\Theta = (T - T_0)/(T_h - T_0)$; $\mathrm{Pe} = h_0\langle u_0\rangle\rho_0 c_0/\lambda_0$ is the Peclet number for the inlet conditions; $C_V = \rho c/\rho_0 c_0$ is the dimensionless value of volumetric specific heat; $K = \lambda/\lambda_0$ is the dimensionless thermal conductivity (C_V and K depends only on θ).

Thus, to calculate heat transfer, it is necessary to specify the form of the function f(z) and find the solution of Eqs. (9) and (13) with the following boundary conditions:

$$\Theta = 0, \; H = 1 \quad \text{at} \quad X = 0;$$
$$\partial\Theta/\partial z = 0, \; f = 0 \quad \text{at} \quad z = 0;$$
$$\Theta = 1, \; \partial f/\partial z = 0 \quad \text{at} \quad z = 1.$$

Method of Solution. The velocity profile f(z) is approximated by a fourth-degree polynomial

$$f(z) = tz + az^2 + bz^3 + dz^4. \tag{14}$$

Using the boundary conditions and the values for γ and β, we obtain the system of equation

$$\left.\frac{df}{dz}\right|_{z=0} = \beta, \quad \int_0^1 f dz = 1, \quad \int_0^1 f^2 dz = \gamma, \quad \left.\frac{df}{dz}\right|_{z=1} = 0,$$

the solution of which gives values of the coefficients of the polynomial, expressed through β and γ, in the form

$$t = \beta,$$
$$d = (255-45\beta \pm \sqrt{252000\gamma-309375+8250\beta-375\beta^2})/16,$$
$$a = 6-5\beta/2 - 4d/5, \quad b = 4(\beta - 7d/5-3)/3.$$

Since Eq. (14) should be transformed into a second-degree polynomial (Poiseuille profile) with the values γ = 1, 2 and β = 3, a minus sign should be used in front of the radical in the expression for d. The values of γ and β are determined by approximating experimental data from measurement of the velocity profile in a film [2].

The thickness of the film H(X) for Eq. (13) was calculated from Eq. (9) by the Runge–Kutta–Feldberg method [6]. Considering that the increase in the temperature of the boundary layer of liquid at the film drain at the end of the heating zone is no more than 20°C greater than its value at the inlet, we took the viscosity of the liquid in Eq. (9) to be equal to its value at T_0. This allows us to deterine H(X) independently of Eq. (13).

We solved (13) numerically by an implicit two-layer, six-point finite-difference scheme on a rectangular grid [7],

$$L_j^{i+\frac{1}{2}}\frac{\Theta_j^{i+1}-\Theta_j^i}{\Delta X} = \frac{K_{j+\frac{1}{2}}^{i+\frac{1}{2}}(\Theta_{j+1}^i-\Theta_j^i+\Theta_{j+1}^{i+1}-\Theta_j^{i+1})-K_{j-\frac{1}{2}}^{i+\frac{1}{2}}(\Theta_j^i-\Theta_{j-1}^i+\Theta_j^{i+1}-\Theta_{j-1}^{i+1})}{2\Delta z^2},$$

while for the boundary conditions we used the approximation

$$\Theta_j^1 = 0, \quad \Theta_1^i = \Theta_2^i, \quad \Theta_N^i = 1 \quad (j = N \text{ at } z = 1).$$

Here, $L = C_V Pe\ \varepsilon Hf(z)/(1 - X)$; ΔX and Δz are respectively the mesh of the grid along X and z; $X_i = (i-1)\Delta X$; $z_j = (j-1)\Delta z$; $\Theta_j^i = \Theta(X_i, z_j)$ etc.

With frozen coefficients L and K, the above difference scheme is perfectly stable and has a second-order approximation with respect to X and z. The finite-difference equation is solved by the trial run method. Since L and K are dependent on θ at the half-integral nodes, they are found by linear interpolation from their values at the integral nodes. Iteration is used to perform such an interpolation along X. In the first approximation, the values of the unknown coefficients on the (i + 1)-st layer are taken equal to the values of the coefficients on the i-th layer. Then the coefficients L and K are refined from calculated values of temperature on the (i + 1)-st layer and this refinement is continued until the desired accuracy is achieved.

Discussion of Results. A comparative analysis of the results of calculation with constant (for the inlet conditions) and variable values of the thermophysical properties of the liquid [$(T_h - T_0) > 100$°C] showed that the heating deviation may reach 10%. Thus, the calculations must take into account the change in the properties of the solution during heating. Figures 1 and 2 show results of numerical solution of heat transfer and their generalization with allowance for the change in the thermophysical properties of the liquid in relation to temperature. Curves 1-5 in Fig. 1 correspond to the mean-flow-rate heating temperature, while curves 6-10 correspond to the wall temprature for T_0 = 20°C and T_h = 200°C; accordingly, curves 1, 6 were obtained for values Pe $\varepsilon(\alpha$, deg) = 0.7 (30); 2, 7) 2.1 (30); 3, 8) 4.4 (45); 4, 9) 7.5 (20); 5, 10) 12.4 (30). It is evident from the graphs that contact heat transfer in convergent-duct-film units makes it possible to efficiently heat a liquid with relatively low

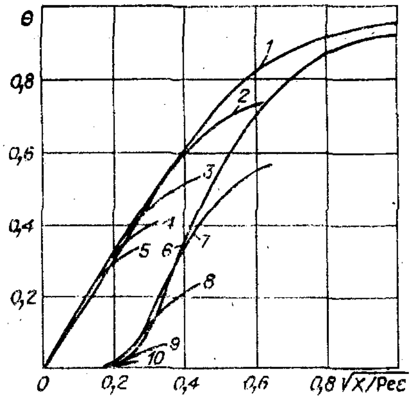

Fig. 1

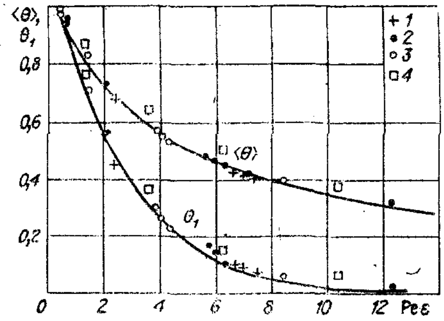

Fig. 2

wall temperatures as a result of variation of the flow-rate, geometric, and temperature characteristics of the process. The main limiting condition in selecting the parameters is the wall temperature at the end of the heating zone (where the film drains from the unit) which is permissible in accordance with scale formation conditions.

Figure 2 shows calculated values of the mean-flow-rate temperature $\langle\Theta\rangle = \int_0^1 f(z)\Theta dz$ and wall temperature θ_1 for different angles of inclination of the generatrix of the convergent duct relative to the horizontal with X = 0.8. Points 1-3 correspond to angles of inclination of 20, 30, and 45°, while the lines are drawn from the approximate expressions $\langle\Theta\rangle = 1{,}43 \exp[-0{,}5\,(\mathrm{Pe}\,\varepsilon)^{0{,}45}]$, $\Theta_1 = 1{,}19 \exp(-0{,}38 \times \mathrm{Pe}\,\varepsilon)$. These expressions simplify the problem of selecting the regime and geometric characteristics of the heat exchanger.

The applicability of the model examined here was evaluated by conducting experiments on a convergent-duct-film unit (α = 45°, ℓ_0 = 38.5 mm) with the heating of water (T_0 = 19°C) by saturated vapor at atmospheric pressure. The results are shown in Fig. 2 by points 4 for film Reynolds numbers of 30, 60, 90, and 120 at the inlet of the unit. The deviation of the theoretical values from the experimental results is due to the effect of the drain hole on the hydrodynamics of film flow of the liquid, which in turn stems from the relatively small dimensions of the convergent duct. This effect weakens with a decrease in the Reynolds number, and the experimental values of wall temperature and mean-flow-rate temperature approach their calculated values.

The above comparison allows us to recommend the model of contact heat exchange examined here for the design of convergent-duct-film heaters for highly mineralized water.

LITERATURE CITED

1. L. I. Sen', Yu. V. Yakubovskii, V. V. Permyakov, N. G. Polishchuk, and V. G. Dobrzhanskii, "Method of heating a salt-bearing liquid," Byul. Izobr., No. 29 (1980).
2. A. M. Te and L. I. Sen', "Film flow of a liquid over the surface of a convergent duct," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, 1, No. 3 (1983).
3. L. I. Sen', "Film heat exchanger," Byul. Izobr., No. 3 (1982).
4. A. M. Te, L. I. Sen', and A. M. Podsushnyi, "High-temperature heating of sea water in a convergent-duct-film unit," in: Marine Engines, Interinstitute Handbook [in Russian], DVGU, Vladivostok (1981).
5. L. I. Sen', A. M. Te, and O. Yu. Tsvelodub, "Model of film flow of a liquid over the surface of a convergent duct," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1983).
6. J. Forsythe, M. Malcolm, and K. Moler, Machine Methods of Mathematical Computation [Russian translation], Mir, Moscow (1980).
7. V. M. Paskonov, V. I. Plezhaev, and L. A. Chudov, Numerical Modeling of Heat and Mass Transfer Processes [in Russian], Nauka, Moscow (1984).